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Ideal distortion-less bending of a focused non-paraxial electron beam

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ABSTRACT

In specific applications, where electron beam is required to be bent after passing through the focusing optics, the beam spot on the target is distorted due to asymmetry introduced by the bending. For complete elimination of the above distortion, a circularly asymmetric initial velocity distribution of electrons that move in a radially decreasing magnetic field (i.e. index of field gradient = 1) is proposed. The expression is quantitatively exact for beams with any angle of bending even when paraxial condition is not satisfied i.e. beam cross-sectional diameter is comparable with the radius of curvature of bending. Modified expression for the case, when kinetic energy of the electrons increases due to time-varying field during its motion through the bending magnetic field is also given as a result of which complexity of the magnetic field design (for strong focusing) can be avoided. Finally, a numerical model of electron gun is utilized to compare the results of proposed model.

1. Introduction

Electron guns are being increasingly used in physical vapor deposition (PVD), welding, accelerators, electron microscopes and spectroscopic studies of metals with high boiling point [1,2]. In all these cases, very high-power density ($>20\,\mathrm{kW/cm^2}$) is attained on the target by tightly focusing the electron beam using an electric or magnetic lens. In addition to this, if the electron gun is placed along the line-of-sight from vapor emitting zone, a part of the generated vapor, ions (created by Saha ionization and electron impact ionization) and bursts of gases due to outgassing proceed towards the cathode–anode region of the electron gun causing electrical discharges [1] or instabilities [3]. So, the electron gun is normally kept in a geometrical shadow region of vapor source. Thus, bending of the electron beam becomes necessary so that it hits the target while maintaining the angle of incidence preferably at $\sim 90^\circ$ to ensure minimal backscattered e-beam power [1]. As the energy of electrons varies from few tens to $\sim 200\,\mathrm{keV}$ (for electron beam welding guns) and background pressure deteriorates by the generated vapor, bending and beam shaping within the vapor by electric field becomes very difficult due to possibility of electrical discharges. So, the only alternative that remains for the bending of electron beam immersed in vapor is magnetic field. In addition to this, simultaneous bending and focusing of the beam of charge particles is a necessity in many electron microscopes and accelerators [4].

In the currently used commercial electron guns for evaporation, a uniform magnetic field is applied for bending of e-beam by 90° or 270°. In these guns, bending of the e-beam automatically focuses it along one-dimension only i.e. along the line of intersection of the plane-containing beam and plane of target. This is possible only for angles of rotation 90° and 270°. But, beam width along the perpendicular direction remains same or becomes more as no focusing action occurs in this direction during bending. In other words, the beam spot on the target when ideally focused becomes a straight line (in reality it becomes an ellipse). On the other hand, if a beam with circular cross-section is initially made convergent by a coaxial magnetic lens, its focusing quality and circular symmetry is distorted by the bending magnetic field through which it has to pass through before hitting the target. This is due to the fact that if initial converging angle is such, beam converges along magnetic field (say X-axis) after bending just at the target, it converges along Y-axis before/after the target. Thus ultimately, we again get an elliptical spot on the target.

However, to achieve very high-power density especially as required by applications like welding, PVD etc., two-dimensional focusing along two perpendicular directions on the target plane is desirable so that the required power density can be achieved at lower total e-beam power. In addition to this, circular symmetry of the e-beam spot on the target is beneficial for welding application and PVD processes where circularly symmetric vapor density distribution is required. Thus, ideally electron beam spot should converge to a point in spite of the bend by the magnetic field.

Implementation of the strong focusing scheme (alternating gradient magnetic field [5]) in an electron beam evaporator or

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wielding machine will be too difficult due to high temperature and presence of metallic vapor in the environment.

For non-paraxial beams when beam cross-sectional diameter is comparable with the radius of curvature of bending, use of weak focusing schemes (used in accelerators) cannot ideally focus all the electrons into a single point on the target even if appropriate bending magnetic field is applied. This is due to the fact that weak focusing of a bent beam requires the beam width to be very small as compared to the radius of curvature of the beam. In this paper, we try to get rid of this limitation of the weak focusing scheme, i.e. ideally focus a finite width (width comparable to radius of curvature) bent electron beam to a single point on target using a unidirectional spatially varying magnetic field. In other words, by focusing of non-paraxial beam we only mean that the beam width is comparable with the radius of bending, but of course the unidirectional magnetic field has to be defect less. Thus, out of the two conditions in conventional weak focusing schemes viz. small beam width and defect free magnetic field, we try to remove the first condition. Second condition may be practically satisfied by choosing a large sized magnet and completely immersing the electron beam from source to target within the magnetic field to avoid fringe field effects at the entrance and exit of dipole. Of course, if the magnetic field defect still persists, the ideal results for finite width bent electron beam predicted by this paper will not be attained.

At first, we derive the theoretical conditions required for focusing a non-paraxial electron beam into an ideal single point on the target for any angle of bending in a magnetic field neglecting space charge. Although, the ultimate limit of the beam width at the focused point will be dominated by space charge effect, we try to eliminate first the imperfections introduced due to bending of the electron beam of finite cross-sectional width. In applications such as electron beam welding machines and evaporation systems, the beam is normally continuous and kinetic energy of the electrons is ~30–200 kV while electron density in the beam is ~10^8/cm^3. In electron microscopes, the electron density is still lower while voltage is ~100–400 kV. In these cases, distortion by bending of the beam plays a significant role and comes into picture well before the dominance of space charge effect which occurs only near the final focal point. We also modify the expression so that it is applicable for the case, when kinetic energy of the electrons increases due to time-varying field during its motion through the bending magnetic field (for applications like betatrons). Finally, we evaluate the performance in a numerical model of electron gun.

2. Theory

Let an electron beam that bends by an angle $\propto$ reaches the target plane after passing through a magnetic field $B(r)$ as shown in Fig. 1. The bending magnetic field $B(r)$ is directed along Z-axis and it varies only with radial distance R from O. Using the differential equation governing the motion of electron in magnetic field [6] radial acceleration is given by

$$\frac{dV_R}{dt} = R\frac{d\phi}{dt}\left(\frac{d\phi}{dt} - \frac{Be}{m}\right)$$

where, m is mass and e is charge of electron.

As $d\phi/dt = V_\varphi/R$, we get,

$$\frac{dV_R}{dt} = R\frac{d\phi}{dt}\left(\frac{V_\varphi}{R} - \frac{Be}{m}\right).$$

So, we see that radial velocity V_R of the electron will remain constant during its travel through the bending magnetic field

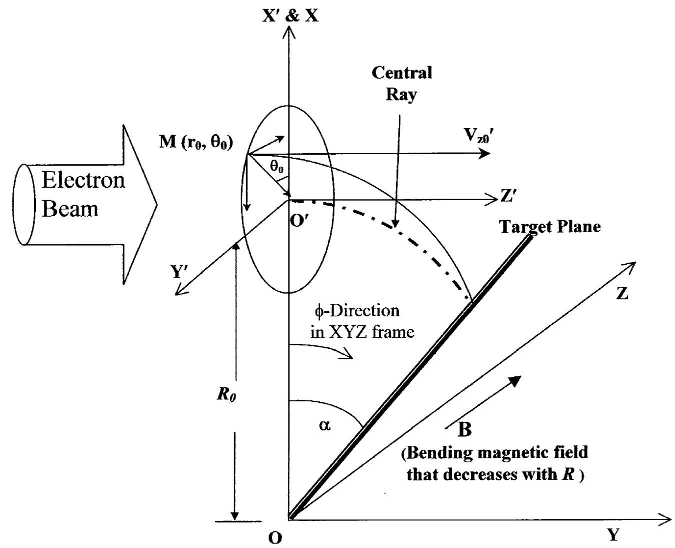

Fig. 1. Three-dimensional geometry of the electron beam for analytical solution.

i.e. $dV_R/dt = 0$ only if,

$$\frac{V_\varphi}{R} = \frac{Be}{m}$$

or

$$B = \frac{mV_\varphi}{eR}.$$

If the above condition is satisfied, the electron moves with constant radial velocity towards O and it has already constant velocity along Z-axis (i.e. along magnetic field). If there is no external source of energy, its tangential velocity must also be same as the initial tangential velocity. Thus we get,

$$V_z = V_{z0},\ V_R = V_{R0} \text{ and } V_\Phi = V_{\Phi 0}. \tag{1}$$

Putting Eq. (1) in expression for B, the required condition for constant radial velocity of electron is given by,

$$B = \frac{mV_{\varphi 0}}{eR}. \tag{2}$$

Thus, if all the electrons in the beam have same initial tangential velocity $V_{\Phi 0}$ and B is given by Eq. (2), then velocities of all the electrons in the cylindrical coordinate system-XYZ shown in Fig. 1 will be conserved. In case of electrons with very high energy (>400 kV), the value of m in Eq. (2) should be taken as relativistic mass of electron instead of rest mass (i.e. $m = m_0/\sqrt{1 - v^2/c^2}$ where m_0 is rest mass, v is speed of electron and c is velocity of light). Because the speed of electron v is constant (as there is no supply of external energy), m is also constant although it is different from rest mass m_0.

Let the height of central ray of e-beam from O is R_0 and an electron is initially at position $M(r_0, \theta_0)$ on plane $X'Y'$. Projection of the trajectory of electron on the plane of curvature is shown in Fig. 2. Since radial velocity of the electron towards central ray of the beam V_r is a vector addition of V_R and V_z both of which are time invariant (Eq. (1)), V_r will also remain same at its initial value V_{r0}. So the electron will converge to the central ray only if,

$$V_{r0}T = -r_0 \tag{3}$$

where T is the total time of travel in magnetic field before hitting the target.

If the electrons have no initial tangential velocity about O' on plane $X'Y'$ i.e. $V_{\theta 0} = 0$, then initial value of radial velocity V_R towards O in XYZ plane will be given by $V_{r0}\cos\theta_0$. As the radial velocity has been made constant, it must be equal to its initial

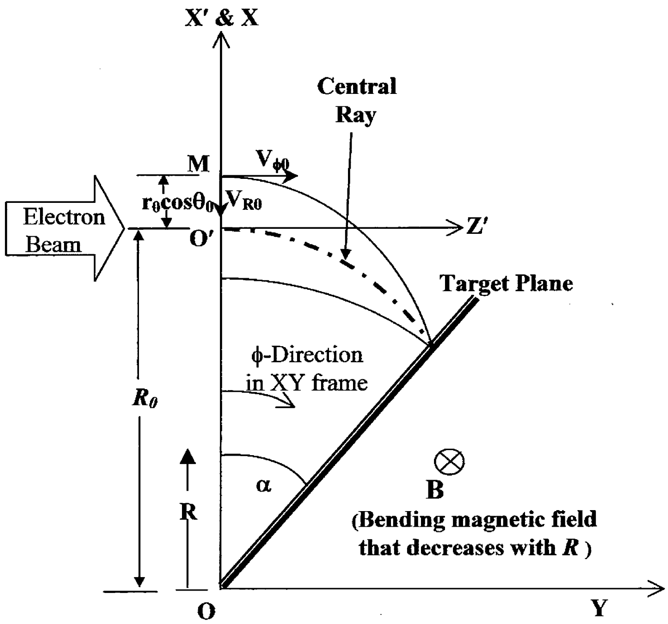

Fig. 2. Projection of trajectory of motion on the plane of curvature of electrons.

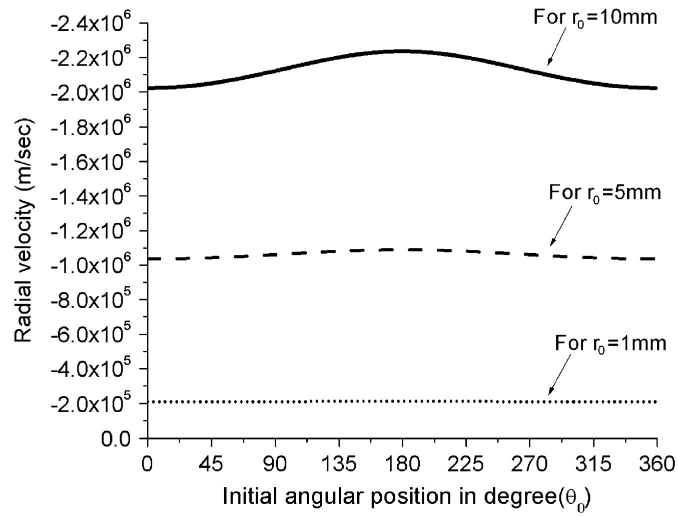

Fig. 3. Variation of radial velocity of electron with initial angular position on $X'Y'$ plane.

value and so,

$$V_R = V_{r0}\cos\theta_0. \tag{4}$$

Now the time of travel of the electron can be calculated from following relation,

$$\alpha = \int_0^T \left(\frac{d\phi}{dt}\right)dt = \int_0^T \left(\frac{V_\phi}{R}\right)dt.$$

As the radial distance from O at any instant of time is, $R = (R_0 + r_0\cos\theta_0) + V_R t$, and using Eqs. (1) and (4), we get

$$\alpha = V_{\phi 0}\int_0^T \frac{dt}{R_0 + r_0\cos\theta_0 + tV_{r0}\cos\theta_0} \tag{5}$$

or

$$T = \frac{R_0 + r_0\cos\theta_0}{V_{r0}\cos\theta_0}\left(e^{\alpha V_{r0}\cos\theta_0/V_{\phi 0}} - 1\right). \tag{6}$$

Putting Eq. (6) in Eq. (3), required initial radial velocity distribution of the electrons within the beam to make them focused after bending by an angle α is given by,

$$V_{r0} = \frac{-V_{\phi 0}}{\alpha\cos\theta_0}\ln\left(1 + \frac{r_0\cos\theta_0}{R_0}\right). \tag{7}$$

Along with the above condition, as stated earlier, initial axial velocity of the beam should be same for all electrons (i.e. $V_{\Phi 0}$ same for electrons), there should be no initial tangential velocity about O' on $X'Y'$ plane (i.e. $V_{\theta 0} = 0$), and of course, magnetic field B should obey Eq. (2). In this way, the effect of beam bending on the quality of focusing can be completely eliminated for any angle of bending even if the beam is non-paraxial.

The radial velocity distribution given by Eq. (7) is circularly *asymmetric* about O'. This is also graphically shown in Fig. 3 where variation of radial velocity of electron with initial angular position on $X'Y'$ plane at different radial distances from O' are plotted (assumed value of $R_0 = 10$ cm and $\propto = 270°$). We see that magnitude of inward radial velocity is highest at 180° position. This asymmetric property (θ_0 dependency) decreases as the radial distance becomes small compared to the bending radius of the beam. This type of distribution may be achieved by use of a circularly asymmetric segmented cathode having varying voltages among the segments for generation of electrons or a stigmator [7] or by use of circularly asymmetric multi-electrode electrostatic lens [8–10]. As the high-voltage component is outside the region of vapor, problem of electrical discharges will not be there.

However, for paraxial beams where beam cross-section is very small as compared to the bending radius of the beam i.e. $r_0 \ll R_0$, Eq. (7) reduces to

$$V_{r0} = \frac{-V_{\phi 0}r_0}{\alpha R_0}. \tag{8}$$

Thus, radial velocity requirement becomes independent of θ_0 i.e. required radial velocity distribution becomes circularly *symmetric* and it is directly proportional to the initial radial distance of the electron from the central ray. So, velocity distribution of Eq. (8) is same as that required for focusing of a straight e-beam on target placed at a distance of '$\propto R_0$' from focus coil. This type distribution can be easily achieved by use of a coaxial magnetic lens [9]. Again due to paraxial nature of the beam, axial velocity of all the electrons remains approximately same. So, the paraxial beam gets focused to a point on the target after being bent by any angle due to presence of only spatially varying bending magnetic field. Of course, similar type of paraxial beam focusing is conventionally known as weak focusing in accelerators.

Now, we modify the expressions so that they are applicable for the applications like betatrons, where kinetic energy of the electrons gradually increases during its motion through the bending magnetic field. For simplicity we will assume that tangential velocity of the electrons V_Φ increases linearly with time at a rate of k i.e. $V_\Phi = V_{\Phi 0} + kt$. Then for constancy of the radial velocity, B should also be increased as per following equation:

$$B = \frac{m(V_{\varphi 0} + kt)}{eR}. \tag{9}$$

Here, electrons pass through the magnetic field in bunches so that at any time all the electrons have same tangential velocity. Although Eq. (9) shows that B increases linearly with time at lower energy (<400 keV), at higher energy relativistic mass of electron has to be taken for m (i.e. $m = m_0/\sqrt{1 - v^2/c^2}$ where m_0 is rest mass, v is speed of electron and c is velocity of light). In that case, B has to be increased non-linearly so as to take care of the increase in m in Eq. (9) as speed of electron v increases with time.

Velocity along Z-axis is not at all affected as the magnetic field is directed along this direction. So, Eq. (5) will be valid in following modified form,

$$\alpha = \int_0^T \frac{(V_{\phi 0} + kt)\mathrm{d}t}{R_0 + r_0 \cos\theta_0 + tV_{r0}\cos\theta_0}. \quad (10)$$

Integrating and using Eq. (3) to replace T (focusing condition)

$$\alpha = \left[\frac{k(R_0 + r_0\cos\theta_0)}{V_{r0}^2\cos^2\theta_0} - \frac{V_{\varphi 0}}{V_{r0}\cos\theta_0}\right]\ln\left(1 + \frac{r_0\cos\theta_0}{R_0}\right) - \frac{kr_0}{V_{r0}^2\cos\theta_0}.$$

By solving the above quadratic equation we get the required initial velocity distribution as,

$$V_{r0} = -\frac{V_{\varphi 0}}{2\alpha\cos\theta_0}\ln\left(1 + \frac{r_0\cos\theta_0}{R_0}\right) - \sqrt{\frac{V_{\phi 0}^2}{4\alpha^2\cos^2\theta_0}\left[\ln\left(1 + \frac{r_0\cos\theta_0}{R_0}\right)\right]^2 + k\left[\left(\frac{R_0 + r_0\cos\theta_0}{\alpha\cos^2\theta_0}\right)\ln\left(1 + \frac{r_0\cos\theta_0}{R_0}\right) - \frac{r_0}{\alpha\cos\theta_0}\right]}. \quad (11)$$

Although, achieving the above initial radial velocity distribution given by Eq. (11) may be practically difficult, we will gain in terms of the reduced complexity of the magnetic field design for perfect focusing of the non-paraxial beam of charge particles. It will be interesting to examine the form of Eq. (11) when, $r_0 \ll R_0$ so that $\ln(1+((r_0\cos\theta_0)/(R_0))) \approx ((r_0\cos\theta_0)/(R_0))$. So, we get,

$$V_{r0} = -\frac{V_{\varphi 0}r_0}{\alpha R_0}\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4k\alpha R_0}{V_{\phi 0}^2}}\right). \quad (12)$$

Thus, in case of paraxial beam required initial velocity distribution becomes circularly symmetric given by Eq. (12) which can be easily realized through a thin magnetic or electrostatic lens.

3. Numerical validation

A numerical model of electron gun was utilized to evaluate the advantages of above mentioned scheme. A computer code using C-programming language was made in-house to trace the electron trajectory in spatially varying magnetic field. We have solved the differential equation of motion using fourth order Runge–Kutta technique. In this model, an electron beam with circular cross-section bends by an angle α during its passage through bending magnetic field to hit the target at 90°. To evaluate the quality of focusing, we calculated the standard deviation 'σ' of their final positions on target from the mean position as used by other researchers [11]. Ideally, a convergent beam without bending should be focused to a point and σ should be zero. Due to bending, non-uniform displacement of the beam along different directions makes the spot elliptical (or a straight line) thereby increasing the σ value. Thus, value of σ signifies how close the beam spot is to the desired ideal case (i.e. a point). We started with about ~5000 electrons uniformly distributed on a circular area at the entry plane of bending magnetic field and the bending angle α was varied from 0° to 270°. In each case, total path length of electron beam in magnetic field was held constant. The value of σ vs. α was simulated for three different cases viz.

(a) Spatially *uniform* bending magnetic field ***B*** and circularly *symmetric* radial velocity of electrons as obtained from a coaxial magnetic lens in the form of Eq. (8) (i.e. conventional scheme).
(b) Spatially *decreasing* ***B*** (Eq. (2)) and circularly *symmetric* radial velocity of electrons as obtained from a coaxial magnetic lens in the form of Eq. (8) (i.e. scheme for paraxial beam).
(c) Spatially *decreasing* ***B*** (Eq. (2)) and circularly *Asymmetric* radial velocity of electrons in the form of Eq. (7) (i.e. exact scheme).

Fig. 4 shows the graph of σ vs. α for above three cases for a beam of cross-sectional radius ~5 mm and path length of ~100 mm. It was found that σ increases very sharply to a maximum of about ~2.8 mm with bending of the beam in case of conventional scheme (case-a) where uniform magnetic field is used. The peak near $\propto = 160°$ is due to the fact that two initially parallel electrons moving in proximity and circular paths become farthest from each other after 180° rotation. In case of exact scheme (case-c), σ is zero for all angles of bending as expected (actually it is ~10^{-5} due to digital error of computation). However, by use of scheme for paraxial beam (case-b) i.e. decreasing B and symmetric focusing, σ was found to be much lower than the case-a which is very good in view of the fact that complexity of cathode or stigmator or asymmetric electrostatic focusing is avoided. To get an idea about the shape of electron beam spot on the target, the simulated e-beam spots on the target for 270° bent beam for the three cases (a, b, and c) are given in Fig. 5. It is clear that shape of the e-beam spot in case of 'b' and 'c' are much better than the shape in conventional case.

4. Conclusion

It was concluded that the effect of bending on the size and shape of electron beam spot on the target can be completely eliminated (theoretically made zero) by choosing a radially decreasing magnetic field from the centre of curvature along with a *circularly asymmetric* radial velocity distribution even when paraxial condition is not valid. In the paraxial case, elimination of the bending effect is possible only by using a circularly symmetric radial velocity distribution that is easily achieved by a coaxial thin

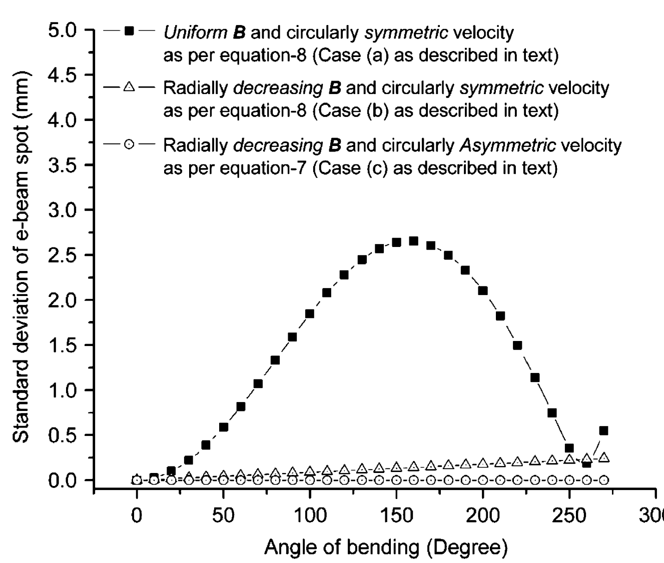

Fig. 4. Variation of standard deviation of electron beam spot with angle of bending for three different cases (a), (b) and (c).

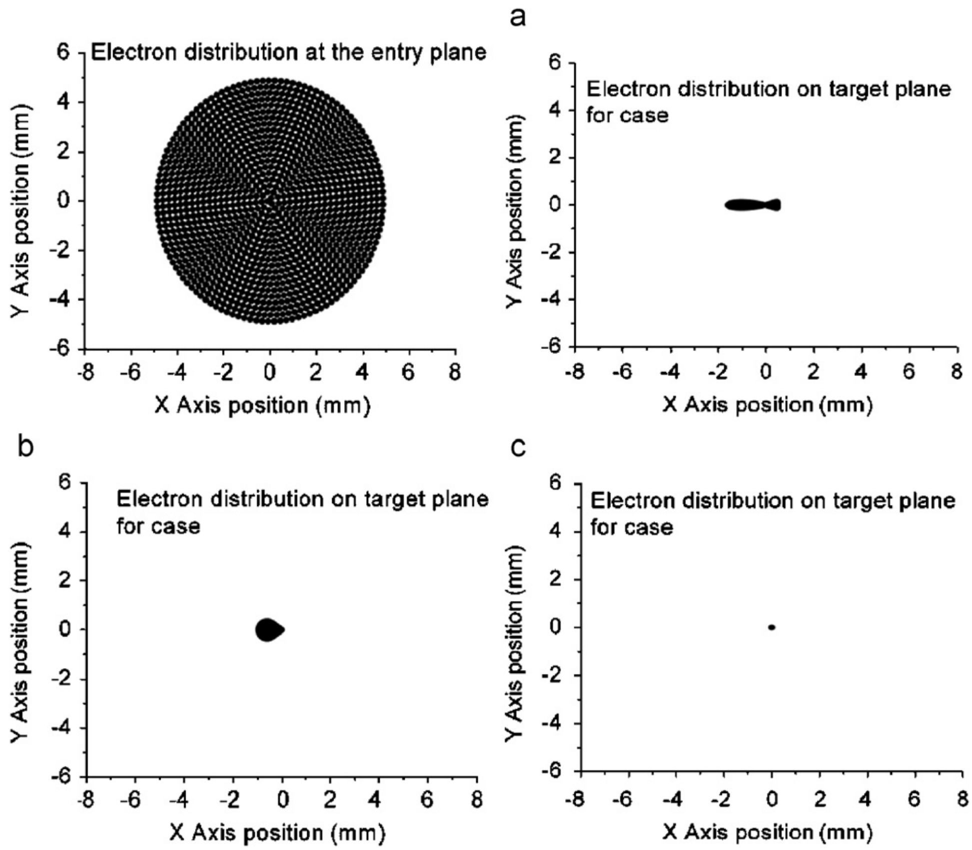

Fig. 5. Variation of size and shape of electron beam spot on target for 270° bending from the initial spot size for three different cases (a), (b) and (c).

magnetic lens. These conclusions are valid for any angle of bending of electron beam. The scheme can also be extended to applications like betatrons where kinetic energy of the particles increases with time during bending. The radially decreasing flux can be obtained by linearly increasing the air gap between the two pole faces. By carrying out this type of modification in the currently used bent beam electron guns, two-dimensional focusing of the electron beam can be achieved leading to a circular electron beam spot with high-power density along with the advantage of keeping electron gun in geometrical shadow region of vapor.

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